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Article

Blind source separation algorithm for biomedical signal based on lie group manifold

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Abstract: Independent Component Analysis (ICA) is a powerful tool for solving blind source separation problem in biomedical engineering. The traditional ICA algorithm ignores the Lie group structure of constrained matrix manifold. In this paper, a gradient descent algorithm on Lie group manifold is proposed based on the geometric framework of optimization algorithm on Riemann manifold. Firstly, the orthogonal constraint separation matrices are regarded as a Lie group manifold, and the gradient of ICA objective function on the Lie group manifold is given by using Riemann metric; Secondly, the geodesic equation of the current iteration point along the gradient descent direction is calculated; Finally, a new iteration point is obtained by moving a certain step along the geodesic line, meanwhile, the step length can be adjusted adaptively. Simulation results show that the gradient algorithm on Lie group manifold is feasible for blind Source Separation, and its performance (convergence speed, stability and error) is better than other algorithms.

Keywords: ICA; Lie group manifold; gradient descent; blind source separation

1. Introduction

Functional studies in molecular biology show that the majority of genes have pleiotropic function. Almost every gene can respond to a variety of distinct external signals. Linear factorisation methods have an intrinsic way to associate a gene (sensor) to several sources of signal (biological functions) which makes it a suitable tool for analysis of complex biological data. Moreover, since the linear factorization methods are based on some kind of averaging of the data (calculating data moments), they are intrinsically more stable to the presence of high levels of noise in the data and partial removal of samples, if compared to the agglomerative clustering methods. Blind source separation refers to the process of recovering each source signal from only partial prior knowledge of the observed signal and the source signal, when both the source signal and the transmission signal channels are unknown. Independent component analysis (ICA) is an effective method for blind signal separation, which is a new signal processing technology in modern times [1–4]. It satisfies the principle of statistical independence, optimizes the objective function through various algorithms, and obtains the estimation components for source signals, which are independent and non-Gauss distribution. At present, ICA algorithms are mainly divided into batch algorithms and adaptive algorithms. Batch algorithms, such as FastICA algorithm [5] and Joint diagonalization algorithm [5], have good numerical stability, but are not suitable for real-time update of observation data. Adaptive algorithms, such as EASI algorithm [6] and Natural gradient ICA algorithm [7], have less computational complexity and online learning ability, but the convergence and stability are greatly affected by the learning step. It is worth noting that under the basic assumption of ICA, the separation matrix is constrained by orthogonality, which is equivalent to the white signal. The traditional ICA learning algorithms do not make use of the fact that the constraint set is a Riemannian manifold, which causes the separation matrix to have a large amount of computation and instability in the iterative process. ICA is an optimization problem with manifold constraints. If this constraint is treated as an equality constraint, the numerical calculation effect may not be very good. Therefore, the nonlinear constrained optimization problem of ICA is transformed into an optimization problem on a Riemannian manifold, which will be very reasonable and efficient.

In recent years, optimization algorithms on manifolds [8–12] have become an important research direction in the field of nonlinear programming, and have been successfully applied in many fields such as pattern recognition, image processing, blind source separation, and biomedical signal processing. Optimization algorithms on manifolds treat constraint sets as manifolds, thus transforming traditional constrained optimization problems into unconstrained optimization problems.The Riemannian manifold optimization framework on manifolds can effectively deal with nonlinear optimization problems with constraints. It can unify constrained and unconstrained models on Euclidean space. Edelman [13] proposed the Newton format and conjugate gradient format on stiefel manifold; Zhang [14] studied the gradient algorithm on stiefel manifold and its application in feature extraction; Song [15] studied the optimization method on Riemann manifold to solve sparsity PCA; Li [16] studied the interference alignment scheme based on the conjugate gradient algorithm on grassmanian manifold, however, the gradient algorithm on manifold for ICA is rarely reported. Based on the geodesic flow tool on Riemannian manifold, this paper proposes a gradient algorithm on Lie group manifold, which has adaptive adjustment of step size and guaranteed orthogonal constraints. We established a line search method on the Lie group manifold and provided a detailed explanation of the determination and calculation method of the descent direction of non-smooth functions on the Lie group manifold in the algorithm steps. This paper also verified the convergence and feasibility of the algorithm through numerical experiments.

2. Materials and methods

This paper deals with the phenomenon that the traditional natural gradient algorithm in signal blind source separation is poor stability and poor separation performance. This paper proposes an improved gradient descent algorithm on Lie group manifold based on the geometric framework of manifold optimization algorithms. Firstly, the orthogonal constraint separation matrices are regarded as a Lie group manifold, and the gradient of ICA objective function on the Lie group manifold is given by using Riemann metric; Secondly, the geodesic equation of the current iteration point along the gradient descent direction is calculated; Finally, a new iteration point is obtained by moving a certain step along the geodesic line, meanwhile, the step length can be adjusted adaptively.

3. Results and discussion

Research has shown that the gradient algorithm on Lie group manifold is feasible for blind Source Separation, and its performance (convergence speed, stability and error) is better than other algorithms. This algorithm not only effectively estimates the mixing matrix, but also has good separation performance for signals with appropriately reduced sparsity requirements.

3.1. Optimization model of ICA

The model for blind source separation of signals is shown in **Figure 1**. Assuming the number of source signals is n, the linear instantaneous mixing model in the determined state is denoted as $s(t) = [s_1(t) \quad s_2(t) \quad \cdots \quad s_n(t)]^T$. After passing through a linear time invariant channel, the source signal $\boldsymbol{s}(t)$ is received by m-receiving terminals and becomes an observed mixed signal $x(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_m(t)]^T$, which are the linear mixture of source signals $s(t)$. Due to the model being in a deterministic state, so $n = m$. The relationship between the $x(t)$ and the $s(t)$ can be expressed as:

$$x(t) = A_{m\times n}s(t) \tag{1}$$

here A is a mixture matrix, each component of $s(t)$ is a random variable with zero-mean.

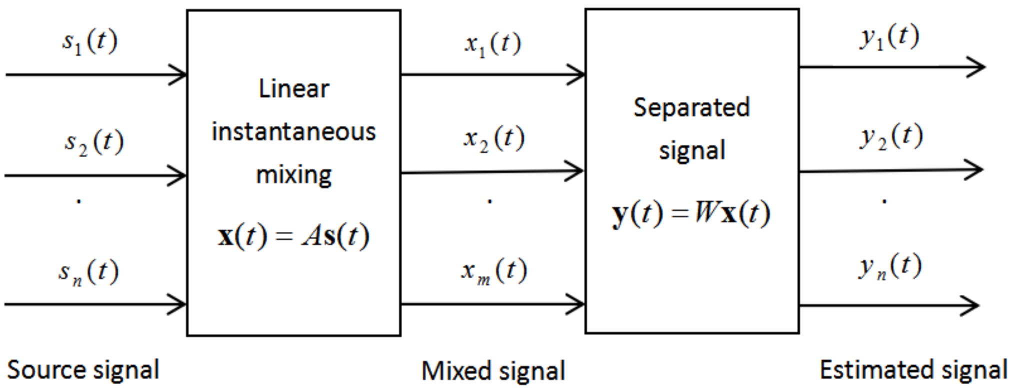


Figure 1. Model for blind source separation of signals.

The purpose of ICA is to find the separation matrix W under the condition of unknow mixture matrix A and unkonw source signals $s(t)$, so as to make each element $y_i(t)$ of the estimation $y(t)$ for source signals $s(t)$ mutually statistically independent.

$$y(t) = W_{n\times n}x(t) = WAs(t) \tag{2}$$

The solution process of ICA is actually an optimization process, which can be realized by data preprocessing, constructing objective function and establishing optimization algorithm.

1) Data preprocessing:

① Centralize observation variables, that is:

$$\bar{x} = x - E(x) \tag{3}$$

② Whitening, looking for linear transformation $z = V_{n\times n}\bar{x}$, where V satisfies:

$$V = D^{-1/2}R^T, E(\bar{x}\bar{x}^T) = RDR^T \tag{4}$$

Whitening can remove the second-order correlation among the components of mixed signal and simplify the mixed matrix A into a new orthogonal matrix $\tilde{A}$, that is

$$E(zz^T) = I, \tilde{A} = VA, \tilde{A}^T\tilde{A} = I.$$

2) Objective function: the probability density of output signal $y(t)$ is $p_y(y, W)$, the probability density of Gauss signal is $p_y(y_{gauss})$, and the negentropy is:

$$J(y_1, y_2, ..., y_n) = \int p_y(y, W)\, ln\, p_y\,(y, W)dy - \int p_y(y_{gauss}) \mathrm{ln} y_{gauss}\, ln\, d\, y = H(y_{gauss}) - H(y, W) \tag{5}$$

3) Optimization mode: because the source signals $\mathbf{s}(t)$ has unit variance, we hope that the estimated signal $\mathbf{y}(t)$ should also have unit variance, that is $E(\mathbf{y}\mathbf{y}^T) = \mathbf{I}$. At the same time, because the observed signal $\mathbf{z}$ is whitened, we get the important property of the separation matrix W which is that the separation matrix W should be orthogonal, that is:

$$E(yy^T) = E(Wzz^TW^T) = WE(zz^T)W^T = WIW^T = WW^T = I \Rightarrow W^TW = I \tag{6}$$

Therefore, the optimization model of ICA problem is a non-convex optimization problem on the orthogonal constraint matrix manifold, that is:

$$\begin{aligned} &\min_{W\in R^{n\times n}} && J(W) \\ &s.t. && W^TW = I \end{aligned} \tag{7}$$

3.2. Gradient algorithm on Lie group manifold

The feasible region in Equation (7) is $O(n, R) = \{W \in GL(n, R)|W^TW = I\}$, which is a Lie group manifold, Its local coordinate system in Euclidean space is $q^1, q^2, ... q^n$, the Riemann metric is $g = g_{ij}\frac{\partial}{\partial q_i} \otimes \frac{\partial}{\partial q_j}$, the tangent vector space of its point W is $T_W O$, so the inner product is:

$$\langle M, N\rangle_W = g_W(M, N) = tr(M^TN), M, N \in T_W O \tag{8}$$

Therefore, the Riemannian gradient of objective function on Lie group manifold is:

$$\begin{aligned} grad_W^O J &= \frac{1}{2}(grad_W J - W(grad_W J)^T W) \\ grad_W J &= \nabla J W^T W \end{aligned} \tag{9}$$

here ∇J is the normal gradient of J on Euclidean space.

Then the geodesic equation at the point W along the direction $H = -grad_W^O J$ on Lie group manifold is:

$$\gamma(t) = WX(t) + QY(t); \\ \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = exp\, t \begin{bmatrix} L & -U^T \\ U & 0 \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{10}$$

here $L = W^T H$ is skew-symmetric matrix, Q, U are QR decomposition matrices of $(I - WW^T)H$.

To sum up, the steps of gradient descent method on Lie group manifold are as follows:

(1) Given initial point W_0, termination constant $\varepsilon > 0$ and initial step size t_0, firstly $k = 0$;

(2) Calculating $grad_W^O J$, if $\|grad_W^O J(W_k)\|^2 = \boldsymbol{g}_W(grad_W^O J(W_k), grad_W^O J(W_k)) = tr((grad_W^O J(W_k))^T (grad_W^O J(W_k))) \le \varepsilon$, stop iteration and output W_k, otherwise enter (3);

(3) The iteration format at W_k along $H = -t_k grad_W^O J$ is:

$$W_{k+1} = W_k X(t_k) + QY(t_k) \tag{11}$$

(4) Adaptive adjustment step size, if $J(W_k) > J(W_{k+1})$, so $k = k+1, t_{k+1} = \alpha t_k, \alpha > 1$, back to step (2), otherwise $J(W_k) < J(W_{k+1})$,so $k = k+1, t_k = \alpha t_k, \alpha < 1$, back to step (3).

The movement of iteration point in gradient descent method on Lie group manifold can be seen in **Figure 2**.

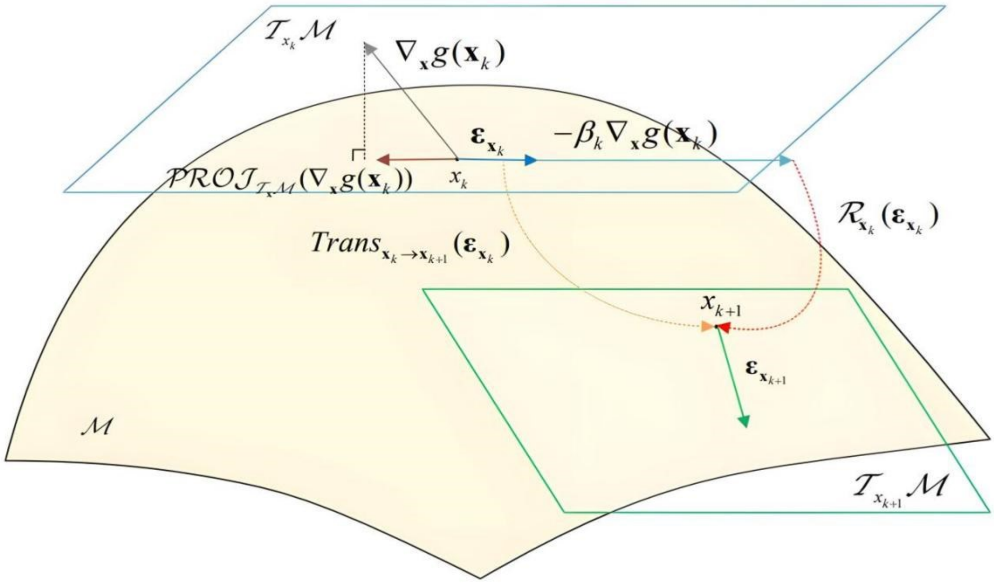


Figure 2. The movement of iteration point in gradient descent method on Lie group manifold.

Under certain conditions, it can be proved that the above gradient algorithm is convergent.

The proof process is long, which can be referred to reference [17].

The brief paragraph for the algorithm steps is in **Figure 3**.

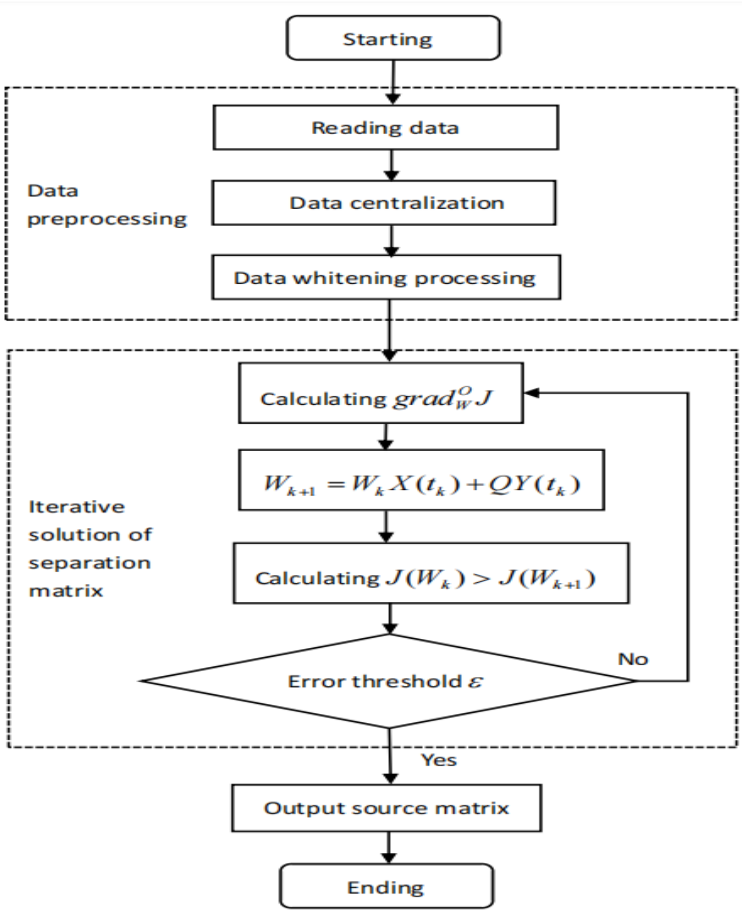
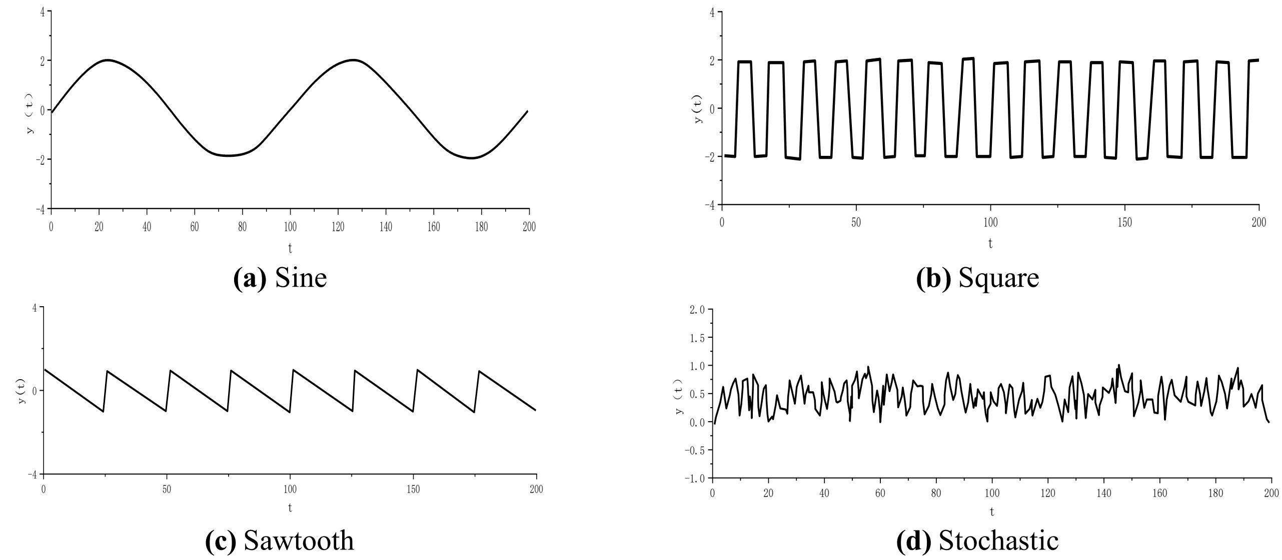


Figure 3. The diagram outlining the algorithm.

3.3. Simulation results

There are four source signals that are sine wave, square wave, sawtooth wave and stochastic wave, see **Figure 4**. All elements of the mixed matrix A meet the uniform distribution of $(-1,1)$, see **Figure 5** for the observed signal.



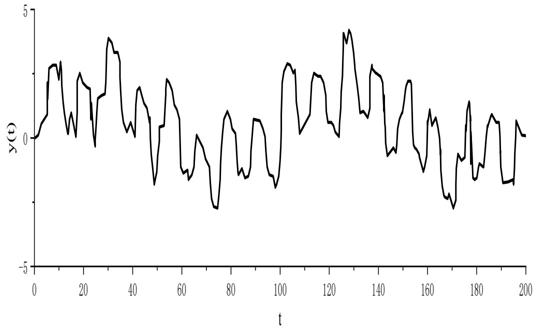
(a) Sine

(b) Square

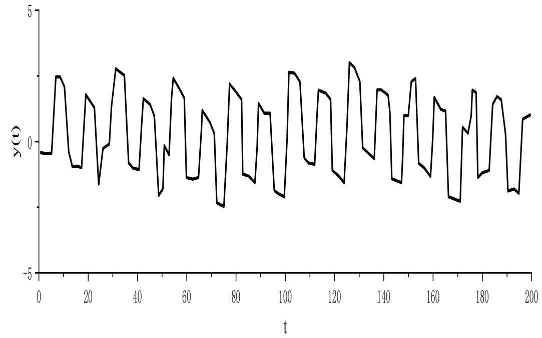
(c) Sawtooth

(d) Stochastic

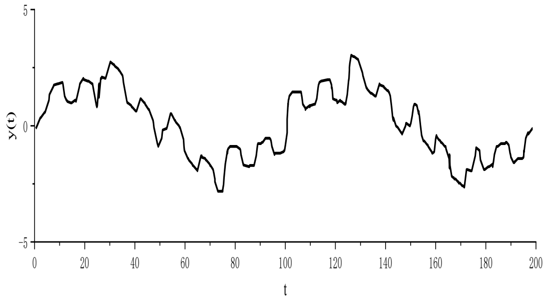
Figure 4. Source signal waveform.



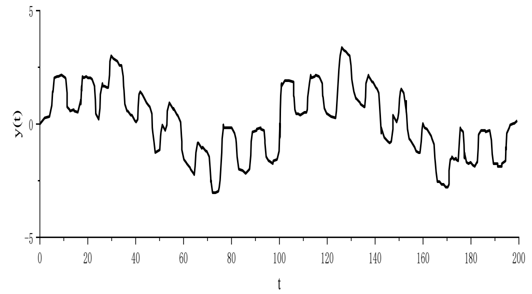
(a) Sine



(b) Square



(c) Sawtooth



(d) Stochastic

Figure 5. Observation signal waveform.

It can be seen from **Figure 5** that the mixed observation signal has been centralized.

The objective function $J(y, W)$ of ICA based on negentropy can be expressed as follows:

$$J(W) = -\frac{1}{2} tr(W^T G W), G = E(xx^T) \tag{12}$$

The corresponding gradients on Euclidean space and Lie group manifold are:

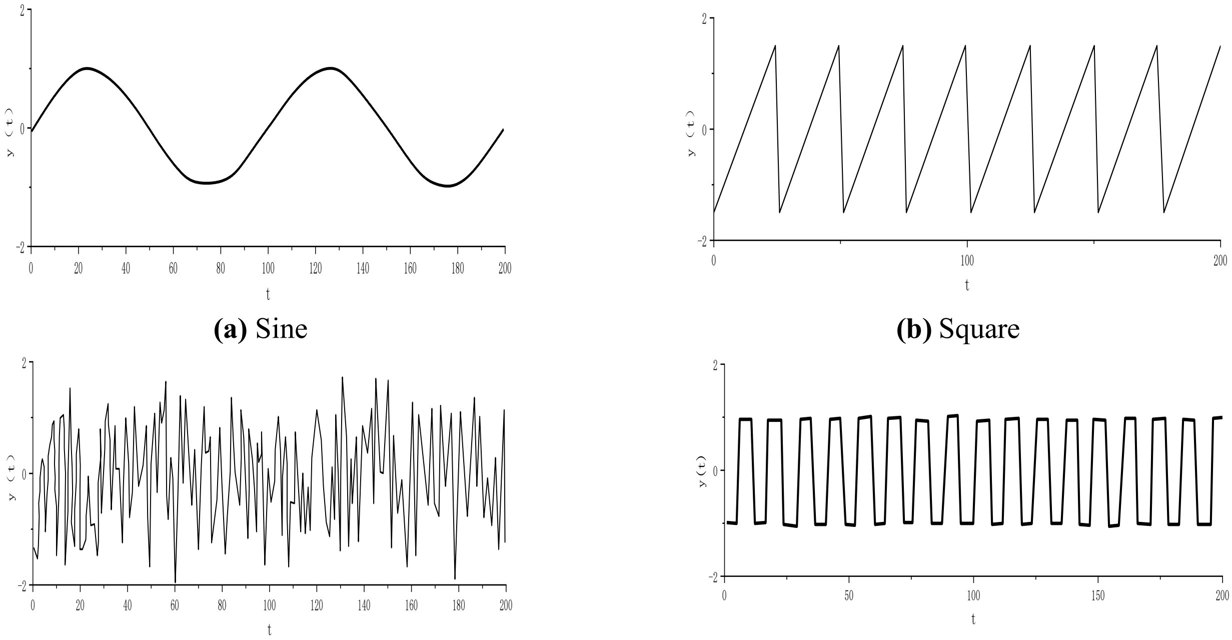
$$\begin{aligned} \frac{\partial J}{\partial W} &= GW \\ grand_W^O J &= GW - WW^T GW \end{aligned} \tag{13}$$

Initial separation matrix is eye $W_0 = I_{4\times4}$, initial step size is$t_0 = 0.0015$, the final optimal solution is obtained by using the iterative scheme Equation (11) on Lie group manifold:

$$W_{1448}^* = \begin{bmatrix} -0.1745 & 0.2621 & -0.7307 & -0.6057 \\ -0.5951 & -0.7904 & -0.1452 & 0.0046 \\ 0.7329 & -0.4642 & -0.4718 & 0.1572 \\ 0.2798 & 0.3017 & -0.4715 & 0.7800 \end{bmatrix} \tag{14}$$

The optimal solution Equation (14) also satisfies the first-order optimality condition of manifold optimization, that is $grand_W^O J(W^*) \approx 0$.

Then the optimal separation signal is shown in **Figure 5**. It can be seen from **Figure 6** that the gradient algorithm on Lie group manifold can separate the mixed signals effectively and accurately, but the order and symbol of signals have changed.



(a) Sine

(b) Square

(c) Sawtooth

(d) Stochastic

Figure 6. Separation signal.

The common evaluation metric for blind source separation algorithms is "crosstalk error" [18]. The performance of algorithm is measured by PI:

$$PI = \sum_{i=1}^{4}(\sum_{j=1}^{4}\frac{|c_{ij}|}{\max_k |c_{ik}|} - 1) + \sum_{j=1}^{4}(\sum_{i=1}^{4}\frac{|c_{ij}|}{\max_k |c_{kj}|} - 1) \quad (15)$$

here $[c_{ij}] = [W \cdot A]_{ij}$, the$PI$is the smaller, the statistical performance of separation algorithm is the better. Comparing the natural gradient ICA algorithm with fixed step size [19] and the simulation degradation algorithm [20], the iteration trend of$PI$is shown in **Figure 7**.

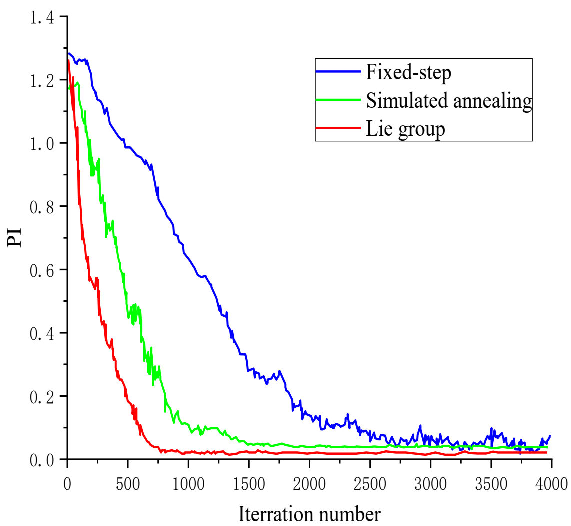


Figure 7. Comparison of PI of three algorithms.

It can be seen from **Figure 7** that the algorithm based on Lie group manifold gradient has faster convergence speed in the early stage of iteration. At the same time, after the iteration is relatively stable, the natural gradient algorithm with fixed step length has discontinuous fluctuation jump, the convergence error of simulation degradation algorithm is slightly larger than that of the gradient algorithm on Lie group manifold, so the algorithm in this paper is more stable and the steady-state error is smaller.

5. Conclusion

The traditional ICA algorithms do not make full use of the Lie group structure of orthogonal constraint separation matrices, and the stability and convergence of algorithm are greatly affected by the learning step. Therefore, this paper introduces the Lie group manifold gradient and adaptive adjustment step. A gradient algorithm framework on the orthogonal group is obtained based on the Riemann manifold optimization algorithm with geodesic. The simulation of blind source separation shows that the gradient algorithm based on Lie group manifold with geodesic to construct iterative scheme, which has faster convergence speed than the natural gradient algorithm with fixed step size and the simulated annealing bionic algorithm, and the stability of algorithm and the accuracy of separation are greatly improved.

Author contributions: Conceptualization, DC and MZ; methodology, DC and MZ; software, DC and MZ; validation, DC and MZ; formal analysis, DC and MZ; investigation, DC and MZ; resources, DC and MZ; data curation, DC and MZ; writing—original draft preparation, DC and MZ; writing—review and editing, DC and MZ; visualization, DC and MZ; supervision, DC and MZ; project administration, DC and MZ; funding acquisition, DC and MZ. All authors have read and agreed to the published version of the manuscript.

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