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Article

Disturbance decoupling for biological fermentation systems with single input single output

Hongjie Li[1], Dandan Wang[2], Mingliang Zheng[2,*]

[1] School of Electronic, Electrical and Unmanned Aerial Vehicle, Anyang Institute of Technology, Anyang 455000, China
[2] School of Mechanical and Electrical Engineering, Huainan Normal University, Huainan 232038, China
*** Corresponding author:** Mingliang Zheng, zhmlwxcstu@163.com

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Abstract: The biological fermentation process has the characteristics of nonlinearity and multivariable coupling. To improve the performance of decoupling control in the fermentation process, a disturbance decoupling control based on the Lie symmetry method is proposed to obtain the analytical feedback control for a class of biological fermentation systems with single input single output (SISO). Firstly, the state-space equations and the disturbance decoupling model for a class of SISO biological fermentation systems are defined; Secondly, the key technologies and algorithm approaches of Lie symmetry theory for differential equations are introduced, and the conditions and the properties of Lie symmetry for nonlinear control systems under group action are given in detail; Finally, the derived distribution of Lie symmetric infinitesimal generators is used to prove the sufficient conditions for local disturbance decoupling in the system, and the closed-loop state feedback analytical law of the system is constructed. The proposed control method is applied to the disturbance decoupling control of mycelium concentration and substrate concentration in the biological fermentation process. Numerical simulation results show that the proposed control method can effectively improve the system decoupling control performance. Meanwhile, using Lie symmetry, the cascade decoupling standard form and the static state feedback law of biological fermentation systems can be constructed.

Keywords: biological fermentation systems; disturbance decoupling; Lie symmetry; state feedback

1. Introduction

The production process of biological fermentation technology is complex. The difficulty of the fermentation cultivation process lies in the large number of process indicators that need to be controlled, high control accuracy requirements, significant mutual influence between various parameters, inability to quickly detect some process parameters, and the unique sterilization operation of fermentation production leading to harsh production environment and high operational risks. Failure to sterilize will result in a large amount of raw material loss and waste pollution problems. At present, most biological fermentation enterprises have outdated technology and equipment, low degree of automation, and unstable product quality. This article takes how to improve the automation control level of biological fermentation production as a research topic, which not only has corresponding practical value, but also has extremely important research significance. A typical biological fermentation system is shown in **Figure 1**: Cu_7S_4 nanocrystal continuous stirred tank reactor.

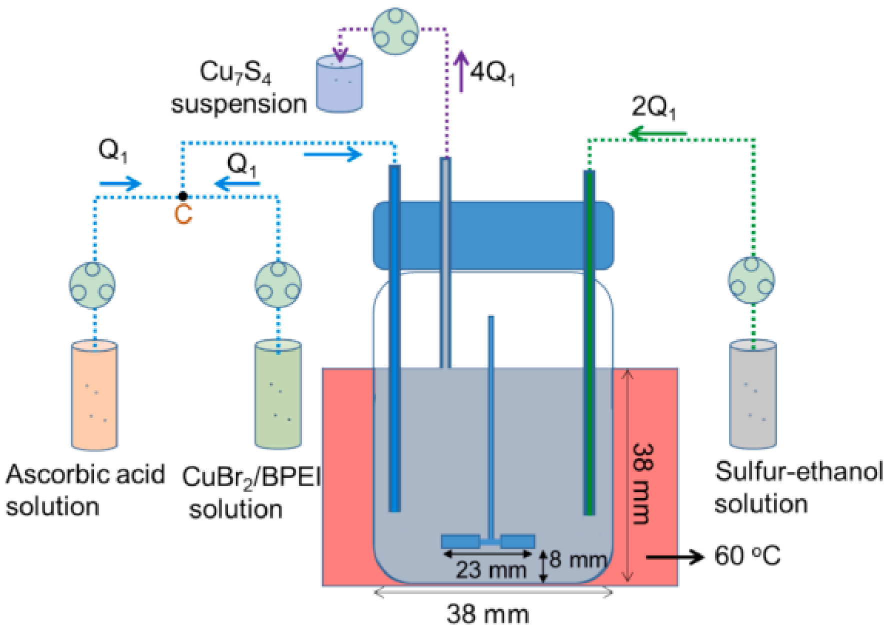

Figure 1. Cu7S4 nanocrystal continuous stirred tank reactor [1].

The biological fermentation process is a time-varying, nonlinear, and uncertain multivariable coupled system that involves the growth and reproduction process of living organisms, and the mechanism is very complex. The fermentation products are not only affected by the physical quantities of the operating environment, but also by the concentration of various fluids during the fermentation process. Physical variables such as temperature and pressure are commonly referred to as operational variables in the fermentation process, while chemical variables such as pH and dissolved oxygen represent environmental factors in fermentation. The biochemical process variables in the fermentation process are bacterial concentration, substrate concentration, and product concentration. Designing robust controllers [2] with anti-interference capabilities for the fermentation process is of great significance for improving the quality and yield of fermentation products. From a mathematical perspective, the finite degrees of freedom nonlinear control systems are usually described in the form of ordinary differential equations in the state space, and its decoupling control [3] refers to using certain control laws such as state feedback to transform the transfer function matrix of the closed-loop system into a diagonal matrix, thereby transforming a coupled multi-variable system into multiple independent single-variable subsystems. In recent years, various control methods [4] have been developed, including modern frequency domain method, adaptive decoupling control method, robust decoupling control, intelligent decoupling control, predictive decoupling, and disturbance decoupling. Among them, the disturbance decoupling control can eliminate the impact of interference on system output through state feedback, so it has a wide range of applications, such as photobioreactors [5] and suspension culture process of animal cells [6], which require the disturbance decoupling. At present, the disturbance decoupling problems of nonlinear control systems are often solved by using differential geometry theory. Reference [7] investigated the necessary and sufficient conditions for almost disturbance decoupling of a class of control systems by using geometric subspace method.

References [8,9] used the concept of relative order in differential geometry to study the necessary and sufficient conditions for disturbance decoupling in MIMO affine nonlinear control systems and time-delay systems respectively, and obtained the specific feedback control laws. Reference [10] studied the disturbance decoupling method of underactuated TORA systems by designing a self-regulating sliding mode compensator. It should be said that the above disturbance decoupling methods have their unique advantages, but there are also obvious limitations, such as not fully utilizing the system's inherent advantages such as symmetries and similarities before designing precise control laws, which makes the conditions for disturbance decoupling strict and the approach complex. Therefore, it is feasible and advantageous to deeply explore the inherent characteristics of the system itself, such as symmetries, and reduce the order of the system, then reduce and linearize the system to achieve disturbance decoupling innovative design of system disturbance decoupling.

As is well known, symmetric nonlinear control systems [11] are widely present in practical engineering, and using the symmetry to study control problems of systems should have certain advantages. At present, some research achievements have been made in the use of Lie symmetry method to study nonlinear control problems. For example, Zhang [12,13] used Lie symmetry to decompose the structure and obtained the controllability conditions of nonlinear control systems, Palazoglu [14] and Karakas [15] first proposed the idea of using symmetry groups to achieve distributed parameter system control, Torres DFM [16,17] used symmetry to study the optimal control problems and obtained the conservation laws of controller. These studies not only enable creative solutions to complex and profound problems in nonlinear control, but also promote further development of Lie symmetry theory. However, there is no mature theory yet regarding the use of Lie symmetry method to study the disturbance decoupling problems in nonlinear control systems. On the basis of the aforementioned literatures, this article discusses the conditions and forms of disturbance decoupling for a class of nonlinear control problems. The specific decoupling approach is given using Lie symmetric infinitesimal generators, and numerical simulations are conducted to illustrate the results.

2. Question formulation and definition

Considering a nonlinear control system with single input and single output as a five tuple $\Sigma(M \times U \times W, M, f, Y, h)$, here f is smooth mapping and satisfies $\pi_M(f) = \pi$, here $\pi_M: TM \to M$ a tangent bundle projection, $\pi: M \times U \times W \to M$ is a smooth fiber bundle, $h: M \to Y$ a smooth mapping, M, U, W, Y are the smooth states, inputs, disturbances and output manifolds respectively. If the coordinates of specified M are $\boldsymbol{x} \in R^n$, the coordinate of U is $u \in R$, the coordinate of W is $w \in R$, the coordinate of Y is $y \in R$, so the local coordinates model of a nonlinear control system is:

$$\begin{aligned} \dot{x} &= f(x, u, w) \\ y &= h(x) \end{aligned} \tag{1}$$

The disturbance decoupling problem [18] of nonlinear control systems refers to finding a static state feedback control law:

$$u = \beta(x) + \gamma(x)v \tag{2}$$

Here, $\beta(x)$, $\gamma(x)$ are one-dimensional smooth functions, v is a new input.

If the output and the disturbance of the closed-loop control system composed of the original system (1) and Equation (2) are completely unrelated, then the original system (1) can achieve disturbance decoupling through state feedback.

3. Lie symmetries of biological fermentation systems

3.1. Lie symmetry for one-order ordinary differential equations

For one-order ordinary differential equations:

$$\frac{dx^\alpha}{dt} = f_\alpha(t,x), (\alpha = 1,2,...,n) \tag{3}$$

Now a continuous transformation Lie group $G = \{T_a | \bar{t} = \phi(t,x,a), \bar{x}^\alpha = \psi^\alpha(t,x,a)\}$, which is a single parameter group, its tangent vector field is $\frac{\partial T_a}{\partial a}\Big|_{a=0} = (\xi(t,x), \eta(t,x))$, and the infinitesimal generator vector $X^{(0)}$ and one-order extension vector $X^{(1)}$ of Lie group G are:

$$\begin{aligned} & X^{(0)} = \xi\frac{\partial}{\partial t} + \eta^i\frac{\partial}{\partial x^i}, X^{(1)} = X^{(0)} + (\dot{\eta}^j - p^j\dot{\xi})\frac{\partial}{\partial p^j} \\ & p^j = \frac{dx^j}{dt}, (i,j = 1,2,...n) \end{aligned} \tag{4}$$

The necessary and sufficient conditions [19] for the form invariance (Lie symmetry) of the one-order ordinary differential Equation (3) under the action of a single parameter continuous transformation Lie group G are:

$$X^{(1)}[\frac{dx^\alpha}{dt} - f\alpha(t,x)\Big|_{\frac{dx}{d_t} - f_\alpha(t,x) = 0} = 0 \tag{5}$$

At this point, the group G is also known as the Lie symmetry group of the Equation (3).

3.2. Necessary and sufficient conditions for Lie symmetry in biological fermentation systems

Now considering the Lie symmetry problem of the original system Σ without interference ($w = 0$). The smoothing effect $\Phi: G \times M \to M$ is a k-dimensional connected Lie group G acts on a state manifold M. If for all $x \in M$, $u \in U$ there are:

$$\begin{aligned} & \Phi_{g*}f(x,u,0) = f(\Phi_g x,u,0) \\ & h(\Phi_g x) = h(x) \end{aligned} \tag{6}$$

So the original control system Σ without interference has the invariance of Lie symmetry (G, Φ).

According to the Lie symmetry theory of the one-order ordinary differential equations mentioned above, the Lie symmetry of the original system (1) without disturbance under keeping the control input unchanged can be described as:

$$\begin{aligned} &X^{(1)}[\dot{x} - f(x,u,0)]\big|_{\dot{x}-f(x,u,0)=0} = 0 \\ &X^{(0)}[h(x)] = 0 \end{aligned} \tag{7}$$

The Equation (7) indicate that the original control system Σ without interference has both Lie symmetry of the state and Lie symmetry of the output.

The Equation (7) are a class of overdetermined partial differential equations for unknown functions ξ, η_i respect to (t, x). Generally speaking, its general solution is very difficult to express, but some specific solutions are clearly solvable. By solving Equation (7), we can obtain the specific Lie symmetry (G, Φ).

Furthermore, by defining the Lie bracket $[,]$ in partial differential operator space $X^{(0)}$, the corresponding Lie algebraic structure $\{X^{(0)}\}$ can be obtained:

$$\left[X_1^{(0)}, X_2^{(0)}\right] = X_1^{(0)}X_2^{(0)} - X_2^{(0)}X_1^{(0)} \tag{8}$$

It should be pointed out here that if the above undisturbed original control system Σ has Lie symmetry (G, Φ), it will have some special properties. Some of its control problems, such as controllability, maneuverability, stability, structural decomposition, decoupling control, no-interaction control and optimal control, which can be fully excavated through its hidden Lie symmetry (G, Φ).

4. Feedback decoupling of biological fermentation systems

The conditions for global decoupling of nonlinear control systems are stronger than those for local decoupling. Not in general, we will only discuss the necessary and sufficient conditions for local decoupling of nonlinear control system Σ. Introducing a distribution Δ_M on the state manifold M:

$$\Delta_M = \mathrm{span}\left\{X_M^{(0)} \middle| X^{(0)} \in T_eG\right\} \tag{9}$$

Here, T_eG is the Lie algebraic space of the Lie group G, span represents a tensor space.

Lemma 1: *If the nonlinear control system Σ without interference has a Lie symmetry (G, Φ), and the smoothing effect Φ is non-degenerate at point q, so the dimension of the distribution Δ_M is k-dimension, it is non-singular involution at point q, and it is an f-invariant distribution contained in the output kernel $Kerh*$.*

Proof. The Δ_M is non-singular involution at the point *q*, please refer to the detailed proof in reference [20]. We only prove that the Δ_M is an *f*-invariant distribution contained in the output kernel $Kerh*$.

Firstly, $h * X_M^{(0)}(x) = \frac{dh(x)}{da}\Big|_{a=0} = 0$, so $\Delta_M \in K\mathrm{erh}*$.

Secondly, $[f, X_M^0] = \lim_{a\to 0}((X_M^0)_{-a} \times f\left((X_M^0)_a x\right) - f(x)\Big)/a = \lim_{a\to 0}((X_M^0)_{-a} \times (X_M^0)_a f(x) - f(x)/a = 0$.

Then, $[f, C_1(x)(X_M^0)_1 + C_2(x)(X_M^0)_2] = L_f C_1(x)(X_M^0)_1 + L_f C_2(x)(X_M^0)_2 \in \Delta_M$.

So it can be inferred that the distribution Δ_M is *f*-invariant by inductive method. □

In summary, the Lemma 1 has been proven.

Theorem 1. *If the nonlinear control system Σ without interference has a Lie symmetry (G, Φ), and there are the following conditions: (1) The action Φ is smooth and non-degenerate; (2) the state space M diffeomorphism the $(M/G) \times G$, M/G is a $n-k$-dimensional quotient manifold space, that is there exist a smooth cross-sectional mapping $s: M/G \to M$ and a projection mapping $p: M \to M/G$ that make $p \circ s$ is an identity mapping; (3) the vector field functions $f(x,u,w) = f'(x,u) + g'(x,u)w$, then there exists a local coordinate transformation $z = R(x) = (z_1, z_2, \dots, z_n)^T$ at the neighborhood of point $q \in M$, which makes the original control system Σ without interference on the coordinates z to be transformed into:*

$$\begin{aligned} &\dot{z}^{(1)} = f^{(1)}(z^{(1)}, u), z^{(1)} \in M/G \\ &\dot{z}^{(2)} = f^{(2)}(z^{(1)}, z^{(2)}, u), z^{(2)} \in G \\ &y = h(z^{(1)}) \end{aligned} \tag{10}$$

Here, $z^{(1)} = (z_1, \dots, z_{n-k})^T$, $z^{(2)} = (z_{n-k+1}, \dots, z_n)^T$ and the sufficient condition for the disturbance decoupling of nonlinear control system *Σ* with interference at point $q \in M$ through states feedback is the vector field functions $g'(x,u) \in \Delta_M$.

Proof. According to Lemma 1, it can be inferred that Δ_M at point $q \in M$ is a *k*-dimensional non-singular involution distribution, therefore, according to Frobenius theorem, there exists a local coordinate transformation $z = R(x)$ enables the $\Delta_M = \operatorname{span}\left\{\frac{\partial}{\partial z_{n-k+1}}, \dots, \frac{\partial}{\partial z_n}\right\}$, taking $z^{(1)} = (z_1, \dots, z_{n-k})^T$, $z^{(2)} = (z_{n-k+1}, \dots, z_n)^T$, so the original control system *Σ* without interference can be transformed into:

$$\begin{aligned} &\dot{z}^{(1)} = f^{(1)}(z, u) \\ &\dot{z}^{(2)} = f^{(2)}(z, u) \\ &y = h(z) \end{aligned}$$

Because the Δ_M is *f*-invariant, that is $[\Delta_M, f] \subset \Delta_M$, so it can be inferred that:

$$\left[f, \ \frac{\partial}{\partial z_i}\right] = -\begin{bmatrix} \dfrac{\partial f^1(z,u)}{\partial z_i} \\ \dfrac{\partial f^2(z,u)}{\partial z_i} \end{bmatrix} \in \Delta_M \Rightarrow \frac{\partial f^1(z,u)}{\partial z_i} = 0. \, (i = n-k+1, \dots, n)$$

So the vector field functions $f^{(1)}$ can be represented as $f^{(1)}(z^{(1)}, u)$. □

Meanwhile, we note that the Δ_M is contained in the output kernel $Kerh*$, that is $\left[dh, X_M^{(0)}\right] = 0, \forall X_M^{(0)} \in \Delta_M$, so it can be inferred that:

$$\frac{\partial h}{\partial z_i} = 0. \, (i = n-k+1, \dots, n)$$

So the function h can be represented as $h(z^{(1)})$.

Because it satisfies $f(x,u,w) = f'(x,u) + g'(x,u)w$, for the nonlinear control system *Σ* with interference, under the coordinate transformation $z = R(x)$, so it can be transformed into:

$$\dot{z}^{(1)} = f^{(1)'}\left(z^{(1)}, u\right) + g^{(1)'}\left(z^{(1)}, u\right)w, \dot{z}^{(2)} = f^{(2)'}\left(z^{(1)}, z^{(2)}, u\right) + g^{(2)'}\left(z^{(1)}, z^{(2)}, u\right)w$$
$$\dot{z}^{(2)} = f^{(2)'}\left(z^{(1)}, z^{(2)}, u\right) + g^{(2)'}\left(z^{(1)}, z^{(2)}, u\right)w$$
$$y = h(z^{(1)})$$

Because $g'(x, u) \in \Delta_M$ is known, so it can be concluded that:

$$g^{(1)'}(z^{(1)}, u) = 0$$

Therefore, the local coordinates $z^{(1)}$are completely independent of interference w, and the output manifold Y is only related to $z^{(1)}$. So, the interference does not affect the output of the system, and the system achieves disturbance decoupling at point $q \in M$.

In summary, the Theorem 1 has been proven.

It should be pointed out that the Equation (10) indicate that a symmetric nonlinear control system can be represented as a series level form consisting of a $n - k$-dimensional quotient space and k-dimensional subsystems, and they are decoupled from each other. If the solution of the quotient space is solvable and the solution of the independent subsystems can be expressed by the solution of the quotient space, so the entire system can be solved.

The coordinate transformation $z = R(x)$ satisfies:

$$\begin{aligned} z_i &= L_f^{i-1} h(x), (i = 1, \ldots, r) \\ z_j &= \lambda_{j-r}(x), (j = r + 1, \ldots, n - k) \\ z_q &= \sigma_{q-n+k}(x), (q = n - k + 1, \ldots, n) \end{aligned} \tag{11}$$

Here, r is the degree of correlation between the state manifold M and the quotient space M/G, $L_f^{i-1}(h(x) = d^{i-1}(h(x))[f(x)]$ is Lie derivative operation, $\lambda \in C^\infty(M/G), \sigma \in C^\infty(M)$.

Finally, we indicate the relationship between the static state feedback control law of the system and the coordinate transformation $z = R(x)$. According to the Equation (10), when $i = r$, regardless of whether i is greater than $n - k$ or not, setting:

$$\dot{z}_r = f_r(z, u) = e(z) + d(z)u \tag{12}$$

So the static state feedback control law can be taken as follows:

$$u = -\frac{e(z)}{d(z)} + \frac{v}{d(z)} \tag{13}$$

Under the feedback strategy of Equation (13), the nonlinear control system Σ can be transformed into:

$$\begin{aligned} &\dot{z}^{(1)} = f^{(1)'}(z^{(1)}, v) \\ &\dot{z}_r = v \\ &\dot{z}^{(2)} = f^{(2)'}(z^{(1)}, z^{(2)}, v) + g^{(2)'}(z^{(1)}, z^{(2)}, v)w \\ &y = h(z^{(1)}) \end{aligned} \tag{14}$$

It can be seen that the output and the interference are decoupled, therefore, by utilizing the Lie symmetries of the system, the feedback disturbance decoupling can be achieved.

5. Simulation example

On state manifold $M = \{x = (x_1, \ldots, x_4) | x_i > 0, i = 1,2,3,4\}$, the Equation (1) of a biological fermentation system on manifold are:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -x_1^2 x_3^{-1} \\ x_2 \\ x_4^{-1} x_3^2 \\ x_1^{-1} x_3 x_4 \end{bmatrix} + \begin{bmatrix} x_1^2 x_2^{-1} \\ x_1 \\ -x_3 \\ -x_4 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w \qquad (15)$$
$$y = x_1^{-1} x_2$$

It is obvious that Equation (15) is an affine nonlinear system, and we use the Lie symmetry method to achieve its decoupling form.

According to the Equation (7), the invariance of Equation (15) under a Lie group satisfies:

$$\begin{cases} \dot{\eta}^1 - \dot{x}_1 \dot{\xi} - \eta^1(-2x_1 x_3^{-1} + 2x_1 x_2^{-1} u) - \eta^2(-x_1^2 x_2^{-2} u) - \eta^3(x_1^2 x_3^{-2}) = 0 \\ \dot{\eta}^2 - \dot{x}_2 \dot{\xi} - \eta^1(u) - \eta^2 = 0 \\ \dot{\eta}^3 - \dot{x}_3 \dot{\xi} - \eta^3(2x_4^{-1} x_3 - u) - \eta^3(-x_4^{-2} x_3^2) = 0 \\ \dot{\eta}^4 - \dot{x}_4 \dot{\xi} - \eta^1(-x_1^{-2} x_3 x_4) - \eta^3(x_1^{-1} x_4) - \eta^4(x_1^{-1} x_3 - u) = 0 \\ \eta^1(-x_1^{-2} x_2) + \eta^2(x_1^{-1}) = 0 \end{cases} \qquad (16)$$

The Equation (16) has a set of solutions as follows:

$$\xi = 1, \eta^1 = \eta^2 = \eta^3 = \eta^4 = 0 \qquad (17)$$

Therefore, the Lie symmetry group corresponding to the system (15) is $G = (0, \ldots, +\infty)$, and the Lie group action Φ on M can be expressed as:

$$\Phi: G \times M \to M, (g, x) \to \Phi_g(x) = gx \qquad (18)$$

Obviousness, $k = dim\, G = 1$, the action Φ is a multiplication operation.

It is easy to verify that the matrix vector $g'(x, u) = [0,0,0,1]^T$ satisfy $g' \in \Delta_M$, so according to Theorem 1, it can be inferred that the original system can achieve disturbance decoupling.

Calculating the correlation between M and its quotient space M/G is $r = 2$. According to the main conclusions Equations (10) and (11) for local decoupling of symmetric nonlinear control systems, we have coordinate transformations:

$$\begin{aligned} z_1 &= x_1^{-1} x_2 \\ z_2 &= x_1^{-1} x_2 + x_3^{-1} x_2 \\ z_3 &= x_3^{-1} x_4 \\ z_4 &= x_1^{-1} x_2 \ln x_2 + \ln x_4 \end{aligned} \qquad (19)$$

Obviousness, $z^{(1)} = (z_1, z_2, z_3)^T \in M/G, z^{(2)} = z_4 \in G$.

Therefore, under the coordinate transformation of Equation (19), the expression of the original system (15) without interference is converted to:

$$\begin{aligned}
&\dot{z}_1 = z_2 \\
&\dot{z}_2 = 2z_2 - z_1 - (z_2 - z_1)z_3^{-1} + (z_2 - z_1)(z_1^{-1} + 1)u \\
&\dot{z}_3 = z_1 z_3 (z_2 - z_1)^{-1} - 1 \\
&\dot{z}_4 = (z_1 + 1)^{-1} z_2 (z_4 + ln(z_2 - z_1)z_3^{-1}) + z_1 + z_1(z_2 - z_1)^{-1} \\
&y = z_1
\end{aligned} \tag{20}$$

Obviousness, the first-three equations of Equation (20) are the manifold of the quotient space, the fourth equation becomes an independent subsystem without any control variables, and it is in series with the quotient space, so the system is locally decoupled.

Furthermore, we study the static state feedback control law, and according to the Equations (12) and (13), we can obtain:

$$u = -\frac{2z_2 - z_1 - (z_2 - z_1)z_3^{-1}}{(z_2 - z_1)(z_1^{-1} + 1)} + \frac{1}{(z_2 - z_1)(z_1^{-1} + 1)} v = \frac{-x_2(x_3 x_1^{-1} + 2 - x_3 x_4^{-1}) + x_3 v}{x_1 + x_2} \tag{21}$$

Setting $v = -x_4^{-1} x_2 + 2x_2 x_3^{-1} - 1$, so the state feedback of closed-loop system is obtained as follows:

$$\begin{aligned}
&\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -x_1^2 x_3^{-1} \\ x_2 \\ x_4^{-1} x_3^2 \\ x_1^{-1} x_3 x_4 \end{bmatrix} + \begin{bmatrix} x_1^2 x_2^{-1} \\ x_1 \\ -x_3 \\ -x_4 \end{bmatrix} (-x_3 x_1^{-1}) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w \\
&y = x_1^{-1} x_2
\end{aligned} \tag{22}$$

Taking the sine wave $w = 0.1\, sin(2t)$ as the interference signal, setting the simulation time is 1s, the initial condition is $x(0) = [10,1,10,1]^T$, simulating disturbance decoupling systems in Matlab, and the waveform of the output response is shown in **Figure 2**. When the initial condition is $x(0) = [10,0.1,10,0.1]^T$, the waveform of the output response is shown in **Figure 3**. Comparing the two graphs, it was found that under different initial values, the interference w did not change the waveform trend of the output response y. So it can be seen that the feedback control law obtained by the Lie symmetry method in this article achieves the disturbance decoupling of the control system.

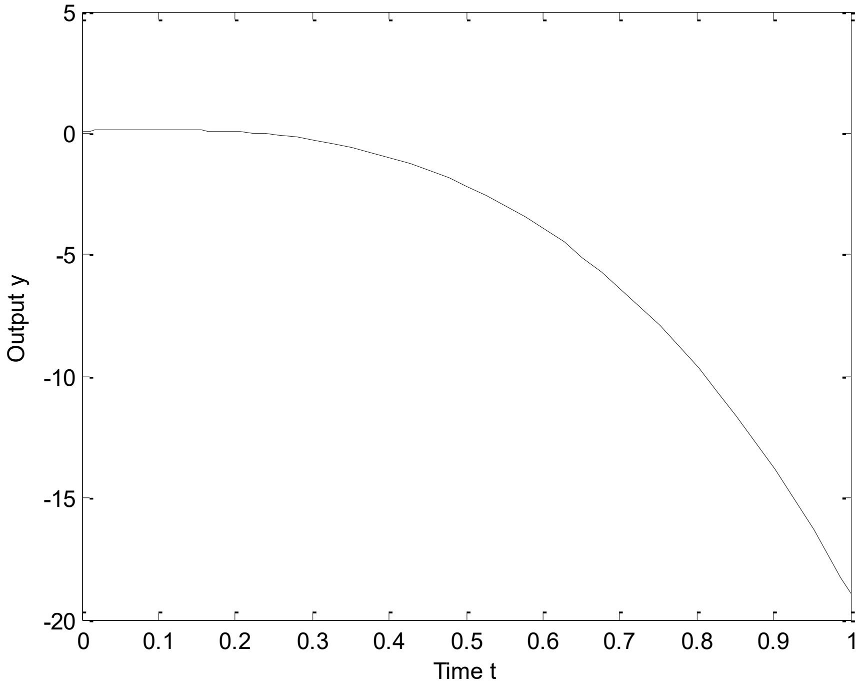

Figure 2. Output response of the system under initial condition. $x(0) = [10,1,10,1]^T$.

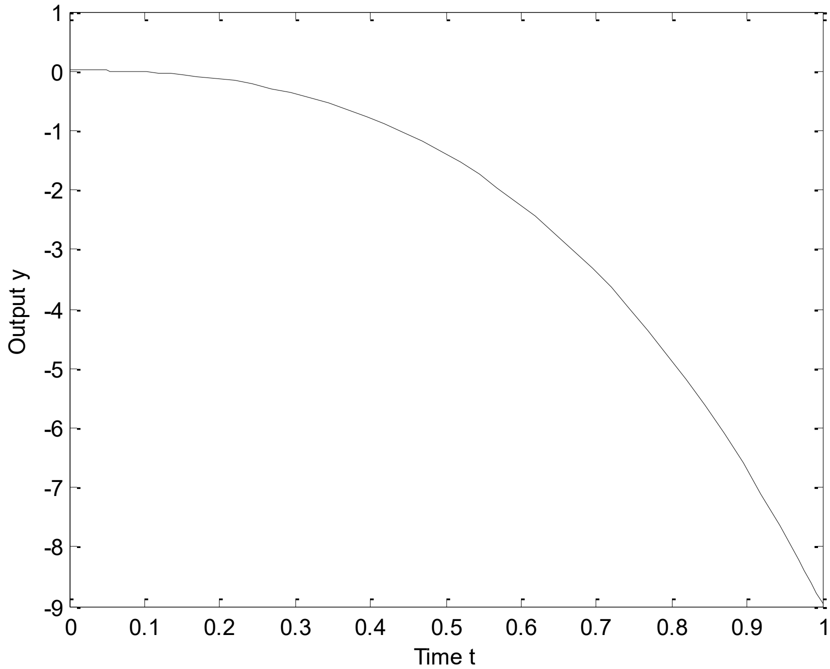

Figure 3. Output response of the system under initial condition. $x(0) = [10,0.1,10,0.1]^T$.

6. Conclusion

Based on the characteristics of fermentation production, analyze and summarize the important process parameters and decoupling control that need to be automatically controlled in fermentation tanks. A feedback disturbance decoupling control based on Lie symmetry method is proposed for a single input single output biological fermentation process with multivariable coupling. The main conclusions are: (1) The sufficient conditions for disturbance decoupling of biological fermentation systems with Lie symmetry only need to verify whether the derived distribution of Lie symmetric infinitesimal generators satisfies the controllability and the compatibility invariant distribution; (2) the disturbance decoupling form of biological fermentation systems with Lie symmetry is equivalent to the quotient system (its dimension is the difference between the system dimension and the Lie symmetry group dimension) and the independent subsystems (its dimension is the Lie symmetry group dimension), which forming a series form. If the solution of the quotient system is solvable, then the entire system is solvable; (3) the static state feedback analysis law of system disturbance decoupling is closely related to the Lie group structure corresponding to Lie symmetry. Overall, the advantage of the Lie symmetry method is that it is theoretically applicable to any complex nonlinear control system, but the difficulty lies in solving the conditional equations that can achieve the Lie symmetry group of the control system, which is a complex and uncertain task. The partial process parameter models of the fermentation process need to be further analyzed and summarized, such as the dissolved oxygen data of the fermentation tank. In the actual production process, there are multiple factors (ventilation, stirring, feeding, defoaming, etc.) that have an impact on it, and the degree of influence of each factor is different. This article does not conduct a systematic analysis and decoupling of dissolved oxygen, and further systematic and comprehensive analysis is needed.

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